Letters to the Editor

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Elastic scattering of fast electrons by nitrogen-14

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It was shown (Fregeau & Hofstadter, 1955) for the nucleus of carbon-12 that the assumption of a half-uniform half-Gaussian shell model gives better agreement with experiment than either the uniform or the Gaussian model. The aim of the present note is to calculate the nuclear form factor for nitrogen-14 on the assumption of a parabolic potential and then to obtain the differential elastic electron scattering cross-section.

The differential elastic scattering cross-section of Dirac electrons (energy E_0) from a target nucleus of mass M, containing Z point protons, each of charge e, is given (Rose, 1961) in the Born approximation by

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_M |F(q)|^2, \tag{1}$$

where $F(q)$ is the form factor and $(d\sigma/d\Omega)_M$ is the relativistic Mott Scattering cross-section for electrons from a massive point nucleus of charge Ze.

$$\left(\frac{d\sigma}{d\Omega} \right)_M = \left(\frac{Ze^2}{2E_0} \right)^2 \cos^2\theta/2 \Big/ \left[\sin^4\theta/2 \left(1+\frac{2E_0}{Mc^2}\sin^2\theta/2 \right) \right] \quad \ldots \quad (2)$$

in which $(1+(2E_0/Mc^2)\sin^2\theta/2)^{-1}$ denotes the centre-of-mass correction, θ being the scattering angle. The form factor is given by,

$$F(q) = \int_v \rho(\vec{r}) \exp(i\vec{q}\cdot\vec{r}) d\vec{r} \quad \ldots \quad (3)$$

and $\hbar q$ is the magnitude of the momentum transfer vector given by

$$\hbar q = \frac{2E_0}{c} \sin\theta/2 \Big/ \left(1+\frac{2E_0}{Mc^2}\sin^2\theta/2 \right)^{\frac{1}{2}}$$

where v, is the nuclear volume and $\rho(\vec{r})$ the charge density, and $\vec{r}$ the radius vector from the centre of the nucleus. The validity of the Born approximation was discussed by Parzen (1950).

For a spherically symmetric charge distribution $\rho(\vec{r}) = \rho(r)$ the integration in (3) may be performed to give

$$F(q) = \frac{4\pi}{q} \int_0^\infty \rho(r) \sin(qr) r dr \qquad \dots \quad (5)$$

If we consider the infinite parabolic well $V \propto r^2$, the nuclear charge distribution is obtained in the analytic form (Buttlar, 1968),

$$\rho(r) = \rho(0)(1+\alpha r^2/a_0^2) \exp(-r^2/a_0^2) \qquad \dots \quad (5)$$

where $\alpha \equiv (Z-2)/3$ and the length parameter a_0 is related to the curvature of the well. Normalization of $\rho(r)$ as

$$\int_0^\infty \rho(r) 4\pi r^2 dr = 1$$

yields

$$\rho(0) = 2\pi^{-3/2} a_0^{-3}(2+3\alpha)^{-1} \qquad \dots \quad (6)$$

The root-mean square radius, weighted according to charge, and defined as

$$a^2 = \int^\infty r^2 \rho(r) 4\pi r^2 dr$$

becomes

$$a = [3(2+5\alpha)/\{2(2+3\alpha)\}]^{\frac{1}{2}} a_0 \, . \qquad \dots \quad (7)$$

Using (5), (6) and (7) in (4) the expression for F is obtained as

$$F = [1-\alpha x^2/\{3(2-5\alpha)\}] \exp[-(2+3\alpha)x^2/\{6(2+5\alpha)\}] \qquad \dots \quad (8)$$

in which $x = qa$,

where we have made use of the integral

$$\int_0^\infty e^{-m^2t^2} \cos nt \, dt = \frac{\sqrt{\pi}}{2m} \exp(-n^2/4m^2). \qquad (m > 0).$$

The value of $x(= qa)$ is varied from 0 to 7 and the form factor calculated in each case. The results are shown in figure 1. It is found that a diffraction zero occurs at $x = 4.3$. It is typical of Born approximation from factors for a charge distribution due to an independent particle shell model of a nucleus for an infinite harmonic well potential The differential scattering cross-section results in the range 30°-90° at an incident electron energy of 400 MeV with the rms radius $a = 2.48 \times 10^{-13}$cm for N^{14} are shown in figure 2. The results of our calculation for the differential cross-section are expected to be reliable excepting the regions in the neighbourhood of the diffraction minimum where the Born approximation is not accurate. In the exact treatment there will be no zero value for the diffraction minimum as obtained in the present calculation.

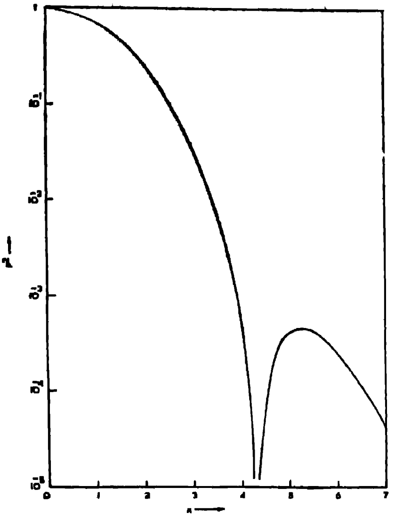

Figure 1. Absolute square of the form factor as a function of X.

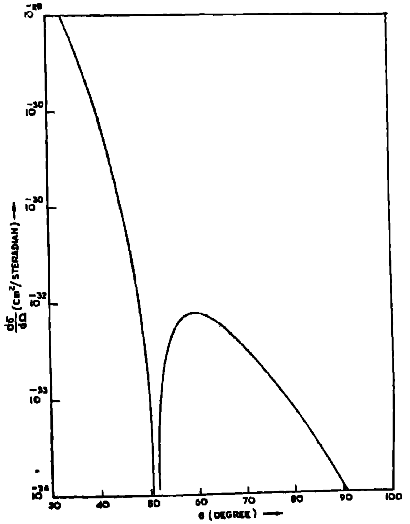

Figure 2. Differential cross-section as a function of scattering angle.

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